Characteristics and applications of precise GPS clock solutions every 30 seconds

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Short title: PRECISE GPS CLOCK SOLUTIONS

Abstract. Using carrier phase and pseudorange data from a small network of globally distributed GPS receivers with precise time references, we are able to estimate GPS clock parameters every 30 sec, with sill.) -(lc'ciIll['tc> [acll[acy, in a computationally efficient manner. Over time and the Earth's surface, there are usually five or more satellites above 15° elevation angle with well determined clock solutions, although occasionally some isolated locations view fewer than four. The accuracy obtained is a factor of 100 to 1000 times better than that of clocks in the broadcast navigation message, and allows post-processing of hi.gll-rate single-receiver kinematic GPS data with few-cr n-level precision when used in conjuncti on with precise GPS orbits. The clock estimates can be interpolated to arbitrary times, with an additional error due to Selective Availability (SA) clock dithering, of approximately 7 cm rms. The interpolation error could be reduced to about 2 cm if 15-SCC data were analyzed instead of 30-sec data. The amplitude of the daily clock variability is typically 24 m for satellites affected by SA. Temporal variations are well modeled by an autocorrelation function $p(d) = \exp(-d^2/2\tau^2)$ with $\tau \approx 106$ sec.

Introduction

Zumberge et al. [1997] describe precise point positioning, that is, the use of GPS satellite orbits and clocks, estimated with data from a global network of geodetic-quality GPS receivers, to efficiently analyze data from arbitrary GPS receivers, without the need to form differenced data wit, h reference receivers. Daily precision of a few mm in the horizontal and cm in the vertical is achievable for stationary receivers.

Because of the intentional dithering of the transmitter clocks as a part of selective availability [e. g., *van Grass and Braasch*, 1996; *Zumberge and Bertiger*, 1996], estimates of them cannot in general be interpolated in time without a severe degradation in accuracy. In particular, the estimates with 5-min spacing produced by most analysis centers in the International GPS Service (IGS), which are accurate to a few cm, are applicable only to data that occur within a few milliseconds of the 5-min times. The application of precise point positioning is thus limited to data that coincides with times for which precise transmitter clocks have been computed.

We describe here an efficient method for the estimation of transmitter clocks at a rate equal to the data rate of terrestrial receivers which view the transmitters, specifically 30 sec for the current global network. We next describe the characterisitics of the high-rate transmitter clocks with a focus on their temporal variations. Finally, we apply precise point positioning with these high-rate clocks to 5-sec geodetic-quality data from both a stationary receiver and a receiver attached to a vehicle.

Method

A typical Jet Propulsion laboratory (J PL) daily analysis of 5-min data from the global network of precision GPS receivers, called the "Flinn solution" [*Jefferson et al.*, 1997], results in estimates of satellite positions as a function of time and the transmitter clock solutions at 5-min intervals. The following receiver-specific parameters are also estimated: (1) spatial coordinates of receiver locations, (2) receiver clock corrections,

(3) zenith troposphere delays, and (4) phase bias parameters. Except for (2) these are either piecewise constant or vary smoothly with time. Knowledge of their values every 5 min is sufficient to determine their values at intermediate times, through interpolation.

If the receiver's time reference is a hydrogen maser (or an especially good rubidium or cesium clock) then the receiver clock corrections will also be temporally smooth. For such receivers the Flinn clock soluti on can be interpolated to yield values at intermediate times. Thus, for the subset of receivers in the Flinn solution with smooth clocks, all parameters except the high-rate transmitter clock solutions are very well known at all times. Data from such a subset can be analyzed at the full 30-sec data rate to estimate precise GPS clock solutions without the need to esti mate any other para meters. This allows data from each satellite to be analyzed without regard to data from any other satellite. (The scheme is analogous to precise point positioning, except the roles o f transmitter and receiver have been reversed.)

The estimation strategy begins with a site selection procedure. The station that was used as the reference dock in the Flinn solution for the day is selected first. (The error for the reference clock is assumed to be zero. Thus all other clock estimates are with resepct to this clock.) Next, clock solutions of other sites used in the Flinn solution are examined for temporal smoothness, as measured by consistency between each 5-min value and an estimate of it basal on cubic interpolation of the four nearest (in time) neighbors. Sites with sufficiently smooth clock solutions 2.5 cm rms are considered as candidates for inclusion. The Flinn site selection procedure is intended to ensure several candidates.

Seven of the candidates, together with the reference clock site, are chosen to give good global distribution. Data from the eight sites at tile full rate of 30 sec are then used to estimate GPS clocks every 30 sec. All other parameters, including phase bias parameters, are fixed to their values as estimated in the Flinn free'-[let~volli solution (again, station clocks and troposphere estimates are interpolated from their 5-minute values to every 30 see), and only GPS clock solutions are estimated< Data from all receivers that view a particular satellite at a given time are used to estimate the single unknown for that satellite and at that time: the satellite clock. It takes 3 to 4 hr on a 40- Mflop RISC workstation to analyze 24 hr of 30-sec data from 8 receivers and 25 satellites.

The 30-sec clock solutions thus determined are consistent with the Flinn free-network orbit, since parameters from that solution, as opposed to the Flinn "fiducial" solution (based on ITRF94 [*Boucher et al.*, 1996]), are fixed when estimating high-rate satellite clocks. A high-rate clock solution consistent with the Flinn fiducial orbit is determined by time interpolation of the slowly varying (hours) difference (order of 10 cm) between the Flinn fiducial and free-llctivork clock solutions. This difference is then added to the just-determined high-rate free-llcltvorti clock solution.

Coverage

Shown in Figure 1 is a map of sites for tile period from August 11, 1996 through April 18, 1997 from which data were used on one or more days in the method described. The labeled open circles indicate sites that were chosen frequently and account for the large majority of data. Solid circles indicate less frequently selected sites. oceanic areas in the southern hemisphere and Antarctica are not well coved, in part because of data latency.

The global coverage varies depending on data availability from sites with good reference clocks. Shown in Figure 2 is a global contour plot indicating the average number of satellites in view above 15° elevation angle with well determined 30-sec clock solutions for January 11, 1997. The eight sites used for that day's analysis are also indicated. This figure indicates a somewhat better than average day.

An example of poorer coverage is shown in Figure 3 for January 17, 1997. On that day only one site in the southern hemisphere with a good clock was used in the Flinn solution. A consequence is that no set of candidates available for the lligll-rate solution can give good global coverage. The resulting geometry limits the solution of lligh-rate transmitter clocks so that the number of such satellites in view above 15° elevation from areas in the southern hemisphere is typically fewer than four.

Accuracy

One consistency check on the solutions is to compare the high-rate results with the Flinn results for common times and satellites. Because all other parameters in the two solutions are identical, differences will arise because only some of the! Flinn stations arc used in the high-rate estimation. The rms difference for this comparison is only 6 mm, consistent with the typical level of post fit phase residuals in either solution.

A second check compares the results from adjacent days near the midnight boundary. For the Flinn solution 30 hours of data centered on noon arc used for a daily analysis, and a six-hour overlap is thus available for each day. For the l[ig}l-rate solutions, 27 hours of data are used, beginning 3 hours before the start of the day of analysis, giving 3 hours of overlap for every clay analyzed. The rms detrended (removal of linear term so that the time series averages zero and has no overall slope) difference between clock solutions during the overlap period is typically 7 to 11 cm for both the Flinn and high-rate solutions. This is consistent with orbit overlaps of 10 to 15 cm.

Availability

The ftp server at sideshow. `jpl.nasa.gov` contains the results of an automated procedure at JPL that produces sp3-formatted files for the Flinn fiducial orbit and high-rate clocks; the area is /pub/ `jpligsac/hirate`. Files formatted for use in JPL's Gipsy/Oasis-II software are in `/pub/gipsy_products/hrclocks`.

Characteristics

Shown in Figure 4 is the detrended (over the day) prn13 clock solution for a four-hour period on January 11, 1997, which exhibits typical behavior for SA-affected satellites. The rms detrended variation over the day is 24 m. The sample autocorrelation function is shown in Figure 5 (small solid circles). The dotted line is the function $\rho(d) = \exp(-d^2/2\tau^2)$ with $\tau =$ 105 sec.

The error made in interpolating the 30-sec solutions to intermediate times can be estimated by asking how cubic interpolation of four equally spaced points with separation δt can predict the value midway bctw'cell the two middle points. For $\delta t = 60$ sec and δt = 120 sec we obtain rms values over the day of 57.4 cm and 405.2 cm, respectively, for the time series in Figure 4. Although we do not have points every 15 see, we can extrapolate the results for $\delta t = 60$ see and δt = 120 sec and estimate that the interpolation error for $\delta t =$ 3(I sec will be about 57.4 cm x $57.4/405.2 \approx$ 57.4 cm $\div 7 \approx 8$ cm at times midway between the 30-sec marks. The midway points correspond to maximum interpolation error; the rms interpolation error for uniformly distributed times is reduced by approximately $1/\sqrt{2}$ to about 6 cm.

Based on some 1-sec clock solutions of *Bertiger et al.* [1997] that were derived from 1-see data, the rms interpolation error for 30-sec spacing is 8.5 cm, reasonably consistent with the above approximation. These same solutions indicated that the interpolation error for 15-sec spacing is 2,3 cm; a significant improvement would thus be obtained if a geographically well distributed subset of the global network, with good reference clocks, were to operate at 15-sec data rate instead of 30 sec.

One other quantity of interest is the instantaneous slope. For this satellite and day its rms value is 22.5 cm/sec, which is not entirely negligible. Consider measurements by two terrestrial receivers simultaneously viewing prn13. For simplicity assume that the receiver clocks are perfect. For common receive time, the difference in phase (or pseudorange) measurements between the two receivers will be independent of prn13 clock error *except* for the fact that signals at common receive times correspond to slightly different transmit times. Measurements from the two receivers are thus associated with slightly different transmitter clock values.

An extreme case occurs when the satellite is at zenith from the point of view of one receiver and at the horizon from the point of view of the other. For this geometry the range differential of $\approx$ 5600 km about 19 ms differential light travel time contributes a residual rms error of about 22.5 cm/sec x 0.019 sec $\approx$ 4 mm. To our knowledge this effect is typically ignored, even in precise analysis of GPS data. For software that accounts for receiver aud transmitter clocks by modeling undifferenced data one can accommodate the effect by estimating instantaneous transmitter clock rates as well as values, provided that the network of receivers gives sufficient geometric strength. For software that forms double-difference datatypes to remove clock errors, the effect is problematic.

Figure 6 shows the detrended prn15 clock solution every 30 sec for a 4-hr period on January 11, 1997. Note that full scale on the vertical is forty-times smaller than that in Figure 4. This is an example of a clock unaffected by SA. The rms variation over the clay is 84 cm, but the variations are smooth on short timescales, so that interpolation of the 30-sec time series is accurate to better than 1 cm rms.

The clock characteristics of the GPS satellites are remarkably consistent, at least for the period August 11, 1996 through April 18, 1997 that was examined. 'l'able 1 summarizes their values and variations.

Applications

When transmitter clocks are available at a high enough rate to track SA variations, then data from a single receiver can be analyzed without the need to from differences with respect to reference receivers. The economy and simplicity of this approach is attractive.

‘1’0 demonstrate the use of high-rate clocks, we apply precise point positioning to 5-sec data from a stationary Trimble SSE receiver in Montpelier, Vermont, which is part of the CORS network (the file README.txt on the ftp server cors .ngs.noaa.gov has additional information), and to 5-sec data from a TurboRogue receiver that was used to track the motion of a vehicle in Australia. Certainly we expect the most interesting applications of high-rate precise clocks to involve kinematic positioning of vehicles equipped with GPS receivers, as well as precision orbit and clock determination of GPS-equipped low-Earth-orbiting satellites. Data from a stationery receiver will provide a best-case scenario for these applications and an obvious truth: no motion.

Stationary receiver

Before invoking the assumption of a moving receiver, we analyze the data with a model that considers the receiver to be motionless. The receiver clock is modeled as white noise, and the zenith troposphere delay is modeled as a random walk with variance time derivative 1 cm^2/hr. Data noise is assumed to be white with amplitude 1 cm for phase and 1 m for pseudorange. No phase ambiguities are resolved. Large a priori uncertainties are assumed for parameters to be estimated. Table 2 indicates the forma] errors of estimated receiver parameters with such a model, assuming both 30-sec and 5-rein data rates. Notice the substantial (and expected) reduction - approximately a factor of 2 to 3 - in the formal errors if one increases the amount of data by a factor of 10.

This reduction is not accompanied by a similar reduction in observed daily repeatabilities of receiver coordinates, as indicated in Table 3, based on 22 single-day analyses. In fact, there is essentially no reduction in daily repeatabilities, indicating temporal correlations in data noise on timescales of a few minutes. For stationary receivers, then, there is little additional information in a data rate that is ten times more frequent than once every 5 min. The more realistic formal errors in Table 2 are those in the 5-min column.

Moving receiver

For a moving receiver, the value in higher rate data is, of course, information on the location of the receiver, whose position will generally change substantially over 5 min. Shown in Table 4 are the formal errors of estimated receiver parameters with a model that allows white-noise variation in receiver location, for both 5- and 30-sec data rates. Because the clock and coordinates are assumed unknown at each measurement, their formal errors are essentially independent of data rate. Because the troposphere is assumed to vary slowly, there is a modest reduction in its formal error for the 5-see data rate.

'l'able 4 indicates strength in the data to determine the 3D position of a moving receiver with few-centimeter accuracy at each measurement. A comparison of the 5-min columns of Tables 2 and 3 suggests optimism in tile formal errors of a factor of 1.86/0.44 $\approx$ 4 for the stationary receiver case. Figure 6 confirms this optimism for the moving receiver, at least qualitatively. Shown there as a function of time is the difference between tile 30-see moving-receiver solution and the 30-sec stationary-receiver solution, for the troposphere and vertical parameters on January 29, 1997. The rms variation over the day is 1.66 cm for the troposphere and 5.74 cm for the vertical. (These include both an average difference ant] variations about the average; the variations are by far the dominant.)

Table 5 summarizes such differences for all estimated parameters, as well as correlations in those differences. The sum in quadrature of the north, east, and vertical components is 6.67 cm, approximately twice the value of the formal error. Nevertheless, the table indicates that kinematic precise point positioning of 30-sec data is achievable with 2- to 3-cm horizontal and 5- to 6-cm vertical precision.

Notable correlations in Table 5 are those among the vertical, clock, and troposphere parameters. (The correlation between the troposphere and vertical is evident in Figure 6.) This is not surprising, since, for a satellite at zenith, these parameters are completely degenerate.

Additional information can of course reduce the degree of degeneracy among these parameters. For example, positioning of a vehicle confined to a track would have only one, not three, degrees of spatial freedom. Or, au independent estimate of troposphere delay from a nearby stationary receiver might be better than an estimate based on data from the moving receiver. A third possibility would be a stable receiver clock to suppress the large correlation - 0.95- between the clock and vertical.

Interpolation effects

Although we indicated the formal errors for 5-sec kinematic positioning in Table 4, the results in Figure 6 and '1'able 5 are based on 30-sec data, for which no interpolation error is present. Shown in 'J'able 6 are the observed rms variations in coordinates relative to the stationary -receiver model. For comparison, the 30-sec results from Table 5 are indicated as well.

The rms error that results from interpolation of points with 30-sec spacing is approximately 7 cm for each transmitter, due to SA as described earlier. Table 4 indicates the sensitivity of estimated receiver position to 1-cm phase data noise. Wc can approximate the interpolation error as data noise, and scaling of '1'able 4 indicates that the estimated 31) receiver position will be 7 x 3.3 cm $\approx$ 23 cm, in reasonable agreement with the 5-see column in '1'able 6. Interpolation of 30-sec precise clock estimates is very much affected by SA.

Vehicle-equipped receiver

The Australia kinematic GPS survey was conducted in Novemebr 1996 as part of a verification and calibration for ariborne radar imagery. An 8-channel TurboRogue receiver was used and its Dorne-Margolin choke-ring antenna was mounted to the roof of a truck. Data were collected at a 1 -see rate and decimated to 5 sec for analysis.

The results are given in '1'able 7, and are consistent with those of the previous section.

Conclusions

Precise estimates of the GPS clocks are routinely estimated every 30 sec using data from a subset of the global GPS network and a computationally efficient algorithm. The clock-dithering aspect of SA results in GPS satellite clock errors that vary by 24 m rms over time scales longer than a few hundred seconds. The variability is well modeled by a Gaussian autocorrelation function with characteristic time of about 106 sec.

Kinematic positioning of moving receivers with 30-sec data rate can be achieved with precision of approximately 7 cm 31) rms in the determination of receiver coordinates. Higher rate data can be analyzed by interpolation of the 30-sec precise clocks, provided one can tolerate a 3 × degradation in ability to determine receiver coordinates. The effect of SA could be largely eliminated in postprocessing applications if a subset of the global network operated at 1.5 sec data rate instead of 30 sec.

Acknowledgments. This work was performed at the Jet l'repulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

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Received 1997; revised 1997

Submitted to the *Navigation*, 1997.

Figure 1. For the period from August 11, 1996 through April 1S, 1997, sites shown as open circles (Algonquin Park, Ontario, Canada; Fairbanks, Alaska; Fortaleza, Brazil; Irkutsk, Russia; Kokee Park, Hawaii; Madrid, Spain; Manama, Bahrain; Tidbinbilla, Australia) and labeled with their 4-character 1 I)s were selected at least 47% of the time and account for more than 70% of the data used in estimating nigh-rate transmitter clocks. Sites shown as solid circles were selected less than 34% of the time and account for fewer than 30% of the data. Improved coverage in oceanic areas and the southern hemisphere would be valuable.

Figure 2. High-rate satellite clocks for January 11, 1997 were determined based on data from the indicated sites. The function represented by the contours indicates the number of satellites above 15° elevation angle with well determined High-rate clocks, averaged over the day. The average value is 5.9 in the northern hemisphere and 6.0 in the southern hemisphere.

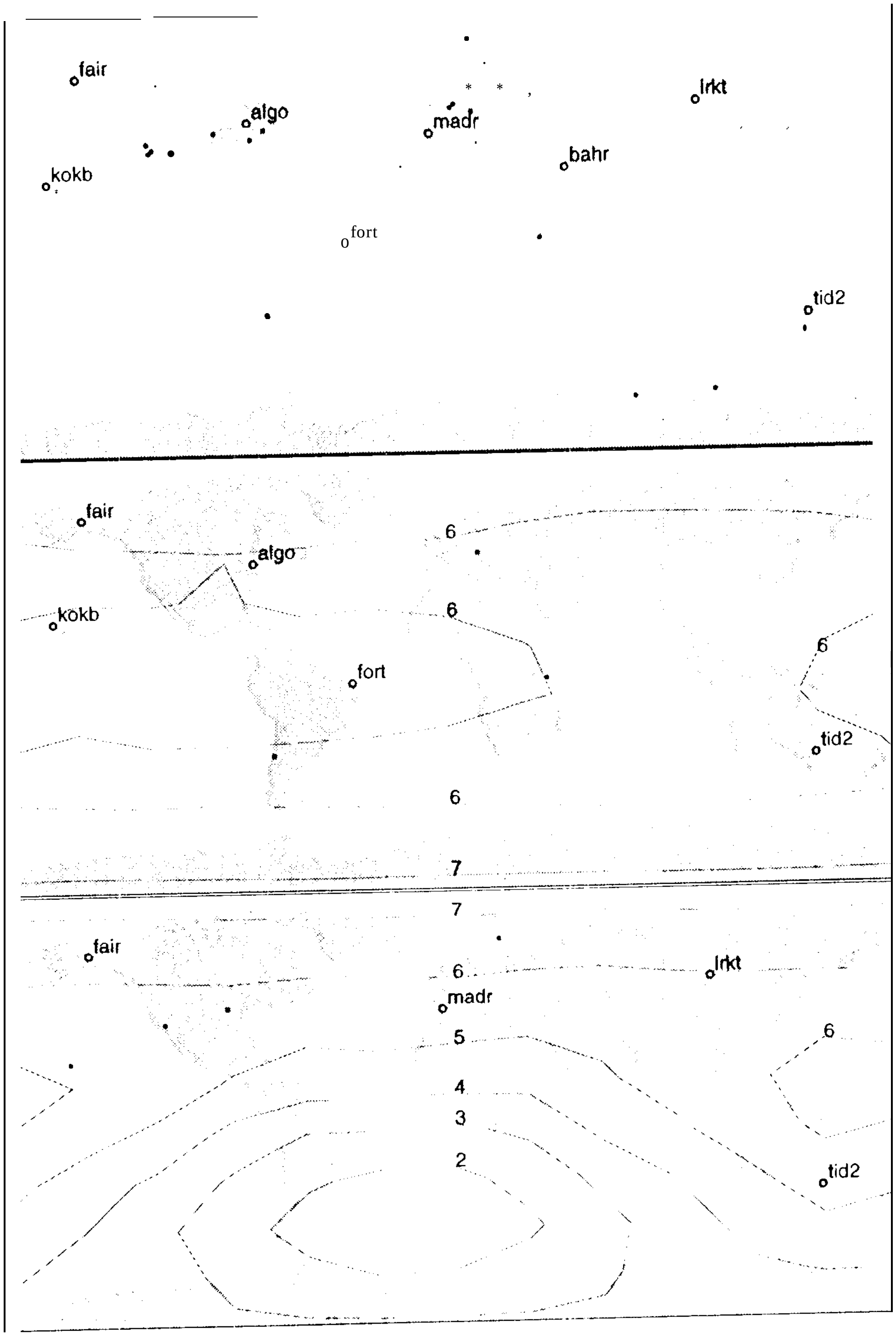
Figure 3. High-rate satellite clocks for January 17, 1997 were determined based on data from seven sites in the northern hemisphere and only one in the southern hemisphere. There ate large regions with only a few satellites with well determined high-rate clocks above 15° elevation angle. Although the number of such satellites averages 5.6 in the northern hemisphere, it is only 3.6 on average below the equator.

Figure 4. The detrended prn1 3 clock solution every 3(I sec for a 4-hr period on January 11, 1997, representing a typical SA-affected transmitter. The rms variation over the clay is 24 m. Cubic interpolation of the 30-sec values is accurate to about 6 to S cm rms.

Figure 5. 'l'he sample autocorrletion based on the detrended prn1 3 clock solution for January 11, 1997. The dotted line is the function $\rho(d) = \exp(-d^2/2\tau^2)$ with τ = 105 sec.

Figure 6. 'l'he detrended pm] 5 clock solution every 30 sec for a 4-hr period on January 11, 1997. Note that full scale on the vertical is forty-times smaller than that in Figure 4; this is a satellite unaffacted by SA. The rms variation over the day is 84 cm. The values can be interpolated to arbitrary times with an rms accuracy of less than 1 cm.

Figure **7.** A comparison of the kinematic solution with the static solution, for 30-sec data.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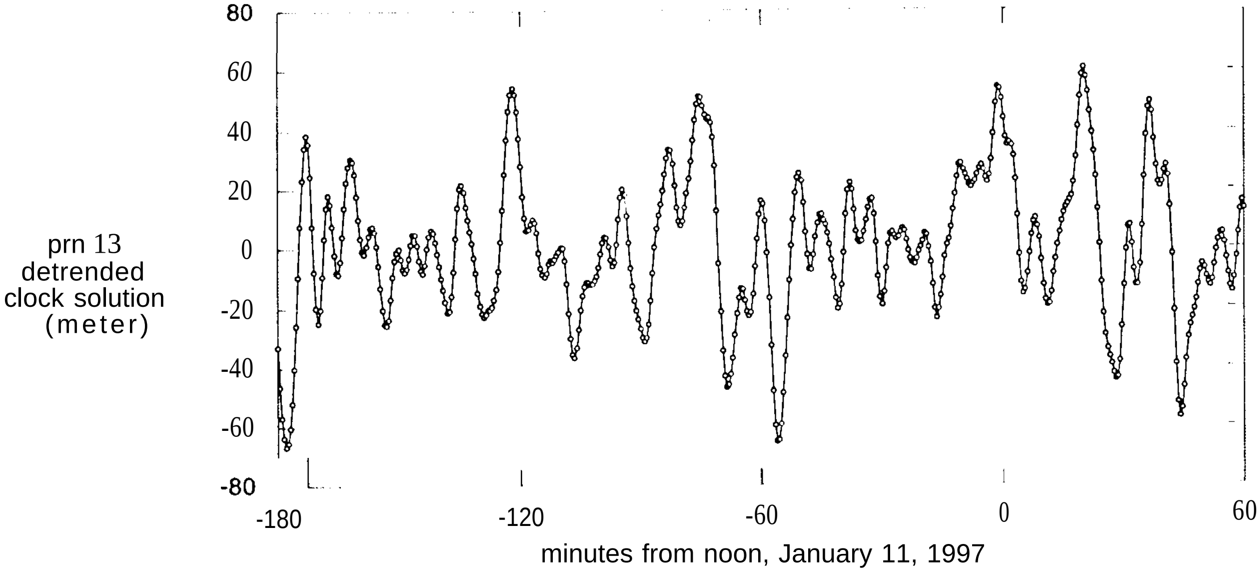
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60
40
20
0
-20
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(meter)
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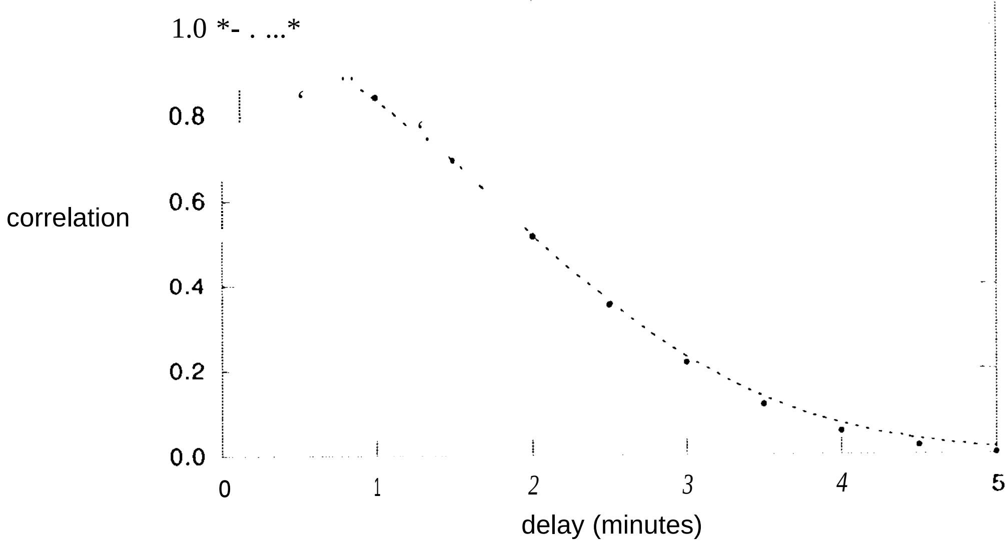
-120
-60
0
60
minutes from noon, January 11, 1997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2
3
4
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delay (minutes)

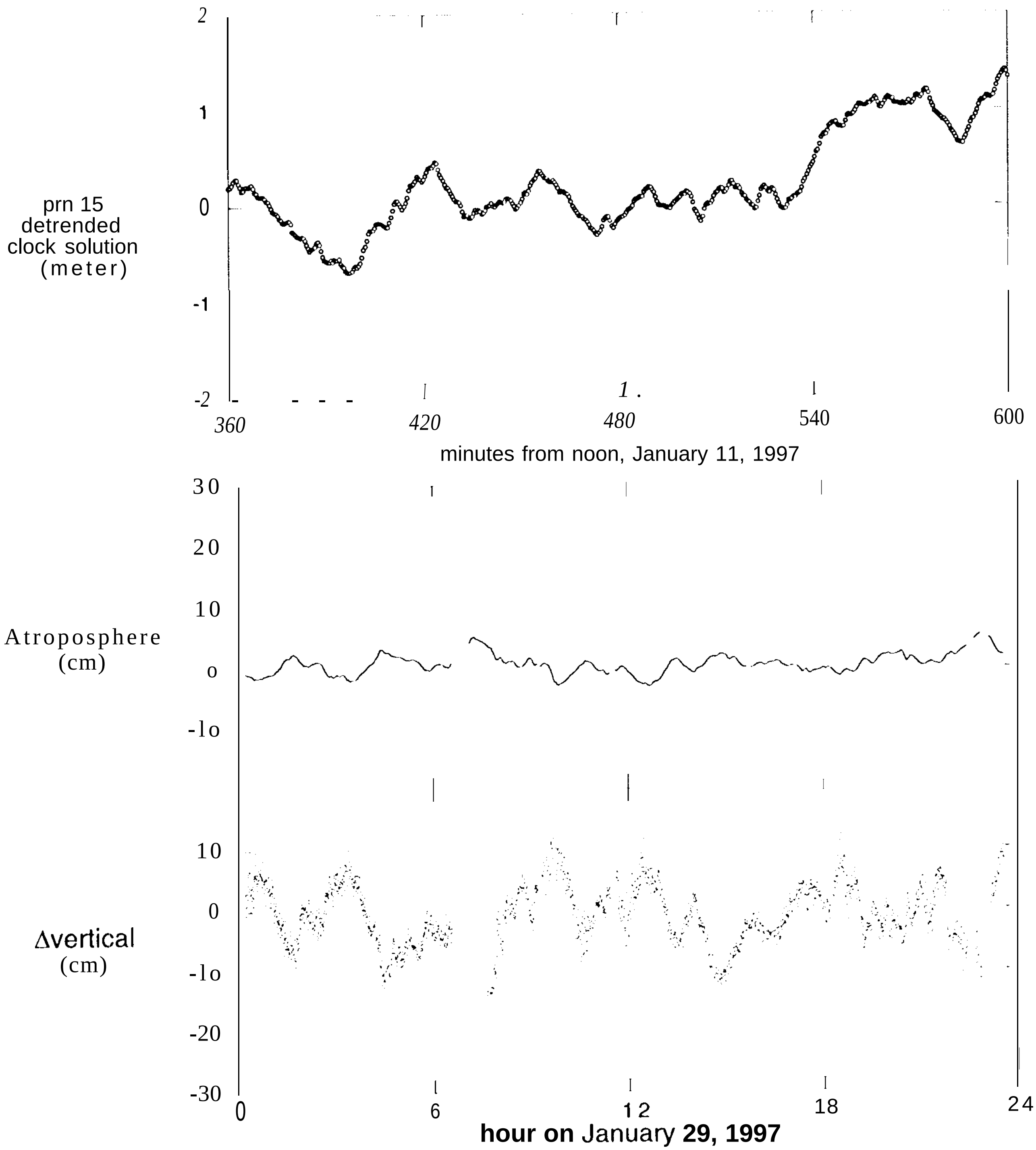
2
1
0
-1
-2
prn 15
detrended
clock solution
(meter)
360
420
480
540
600
minutes from noon, January 11, 1997
30
20
10
0
-10
Atroposphere
(cm)
10
0
-10
-20
-30
Δvertical
(cm)
0
6
12
18
24
hour on January 29, 1997

Table 1. Characteristics of GPS clock variability.

	average	variation
rms amplitude, m	24.2	0.9
τ, sec	105.9	3.9
rms interpolation error, cm	5.7	0.2
instantaneous slope, cm/sec	22.5	0.7

The values are for transmitters affected by SA. The period examined was August 11, 1996 through April 30, 1997 for a total of 6036 satellite days. (About 1% of these were rejected as outliers.) The *variation* column indicates one-standard-deviation variations over days and satellites. The *rms interpolation error* gives the estimated rms error for interpolation of 30-sec points.

Table 2. Formal errors of estimated receiver parameters, assuming a stationary receiver, for 30-sec and 5-rein data rates.

parameter	30 sec	5 min
troposphere, cm	0.19	0.34
clock, cm	0.95	2.56
3D position, cm	0.15	0.44

Table 3. observed daily repeatabilities (receiver position allowed to vary day to clay, but not during any given day).

component	30 sec	5 rnin
north, cm	0.24	0.25
east, cm	0.54	0.50
vertical, cm	1.82	1.77
3D, cm	1.91	1.86

Table 4. Formal errors of estimated receiver parameters, assuming receiver position allowed to vary during the day.

parameter	5 sec	30 sec
troposphere, cm	0.34	0.45
clock, cm	2.4	2.5
3D position, cm	3.4	3.2

Table 5. Variations and correlations in estimated receiver parameters

	troposphere	clock	north	east	vertical
troposphere	**1.66**				
clock	-0.90	**6.35**			
north	-0.02	-0.11	*2.28*		
east	0.12	0.09	-0.30	2.52	
vertical	-0.78	0.95	-0.26	0.20	5.74

The diagonal components in bold indicate the rms difference over the day between the moving-receiver andstationary-receiver solutions, in cm. The off-diagonal components indicate the correlation coefficient.

Table 6. Effect of interpolation error.

component	5 sec	30 sec
north, cm	9.2	2.3
east, cm	7.0	2.5
vertical, cm	20.6	5.7
3D, cm	23.6	6.6

Table 7. Position errors (rms, cm) for several locations in Australia from a kinematic GPS survey using high-rate clocks interpolated to 5 sec.

label	condition	observations	north	east	vertical
cattle	dirt road	422	n/a	n/a	28.5
Eulo	paved road	9	n/a	n/a	4.3
Corner2	paved road	17	n/a	n/a	22.8
Eulo2	paved road	37	n/a	n/a	37.0
Flat tire	stationary	909	8.2	4.7	15.2
Lunch	stationary	938	7.3	9.1	14.5

Vehicle velocities were approximately 60 km/hr and 100 km/hr for the dirt and paved roads, respectively. The *vertical* column indicates the rms residuals between the observed vertical positions from two to four repeat passes along the roads and the best-fit 2nd-order polynomial to these positions. These are not corrected for relative vertical motion between the vehicle and the road, nor for short wavelength road roughness. The horizontal positioning precision can be evaluated for the stationary positions only.